Moving Towards Large(r) Rotors
Is that a good idea?

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Abstract:
We have recently witnessed the development of multi-MW offshore rotors that significantly deviate from the “business as usual” designs. Their common ground: high tip-speed, low solidity but also larger than expected rotor diameter. What is the rationale behind such choices? Which are the pros and cons? These are some of the questions the present paper is addressing. Using classical BEM-based tools we anticipate that these new trends are associated to low induction factor – high swept area rotor designs which may significantly reduce the cost of wind energy produced, in particular, by large offshore wind farms.

Keywords: Large turbines, loads, aerodynamics, blades, offshore

1 Introduction

In order to compare the basic rotor characteristics of these turbines we employed classical similarity rules and upscaled all of them at 10 MW. The results are shown in Table I where the up-scaled version of the 5 MW DOWEC/NREL/UPWIND Turbine [6] is also included. The latter was used as the Reference Wind Turbine (RWT) for the FP6 UPWIND project. It should be noted that the RWT rotor is rather classical with its tip-speed set at the upper range of the onshore design envelop. It is seen in Table I that all other -offshore dedicated- designs are having higher tip-speeds (around 90m/s the ones closer to the market and around 100m/s the two remaining). Higher tip-speed implies higher TSRs and, therefore, reduced solidity. When the rotor tip-speed is no more constrained there are benefits in increasing it for reducing rotor and drive train weights and costs. At the same time thicker high performance airfoils are needed to undertake the aerodynamic loads at smaller chords. Evidently high tip speed is well suited to multi-MW offshore designs. What is surprising however is the large diameter (swept area) that nearly all offshore turbines exhibit as compared to the UPWIND Reference, with the single exception of Sway [5]. Note that the commercial Siemens and Alstom turbines have a 7-10% larger diameter than the expected reference value.

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<tbody>
<tr>
<td>Diameter (m)</td>
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<td>164</td>
<td>190</td>
<td>199</td>
<td>183</td>
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<tr>
<td>Blade Length (m)</td>
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<td></td>
<td></td>
<td>97</td>
<td>89</td>
<td>95</td>
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<tr>
<td>Design RPM</td>
<td>8,56</td>
<td>12</td>
<td>10</td>
<td>8,52</td>
<td>9,39</td>
<td>8,91</td>
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<tr>
<td>Rated Wind Speed (m/s)</td>
<td>11,3</td>
<td>13</td>
<td></td>
<td>13</td>
<td>13</td>
<td>13</td>
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<tr>
<td>Tip Speed (m/s)</td>
<td>80,0</td>
<td>103,0</td>
<td>99,5</td>
<td>88,7</td>
<td>90,2</td>
<td>90,3</td>
</tr>
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Table I: Basic characteristics of several multi-MW rotor designs up scaled at 10 MW using classical similarity rules
Large diameters in this case are not to be confused with the large diameters used in Class II and especially Class III turbines designed to better explore low wind speed sites. Here the focus is on high wind speed offshore sites where Class I (or even harder) wind conditions are normally expected. In a more traditional way of thinking, high-wind-speed designs should rather lead to smaller diameters (like in the Swave case) and certainly not larger. How can the designers combine the high-tip-speed / low solidity option with larger swept areas at severe wind conditions? The rather probable answer is by reducing the specific loading of the rotor. Is this strategy good in view of reducing the cost of wind energy, in particular offshore? This paper will attempt to provide some first answers to this question.

The benefits of increasing the rotor diameter have been studied recently from a wind integration perspective by Peter Molly [7]. He concluded that “there are several reasons why one should start now to optimize the layout of wind turbines in accordance with the requirements of the entire electric supply system and with a view to cost-effectiveness. The result will be a lower rated power of the wind turbines in relation to the rotor disk area”. Although both works are addressing the rotor size, their objectives are completely different. Here the emphasis is on the turbine architecture. Nevertheless our conclusions are complementary as long as a low induction large rotor can not only produce wind energy at lower cost but also contribute to a higher stability level of power capacity available in the grid.

2 Basic Aerodynamic Theory

In order to quantify our analysis we shall use the following set of equations and notation, deriving from Peter Jamieson’s recent book [8], elaborating on Blade Element Momentum (BEM) theory.

We introduce the definitions of the Tip-Speed-Ratio (TSR or \(\lambda\)), of the blade section lift to drag ratio \(k\) and the radius fraction \(x\) according to Eq. (1). Note that \(V\) is the ambient wind speed, \(\omega\) the rotational speed, \(R\) the rotor radius, and \(B\) the number of rotor blades.

\[
\begin{align*}
\lambda &= \omega R / V \\
k &= C_L / C_D \\
x &= r / R
\end{align*}
\]

Assuming a (uniform on the rotor disc) induction factor \(a\), the span-wise distribution of the circumferential induction \(a'\) can be expressed as a function of \(a, k, \lambda \) and \(x\):

\[
a' = -4 \left[ \frac{(2k - 2) x (\lambda - 1/k) - (2k - 2) x + 1}{2k x \lambda} \right] \tag{2}
\]

Using Prandtl’s tip factor

\[
F = \frac{2}{k} \cos^{-1} \left[ \exp \left( -\frac{\lambda - 1}{\sqrt{1 - (\lambda - 1)}} \right) \right] \tag{3}
\]

And introducing the non-dimensional lift distribution \(\Lambda(\lambda, x)\) where \(\sigma(\lambda, x)\) is the chord distribution,

\[
\Lambda(\lambda, x) = \frac{F}{\lambda} \tag{4}
\]

it can be shown that \(\Lambda(\lambda, x)\), which for selected values of \(\lambda\) and \(B\) corresponds to the axial induction value \(a\), is given by Eq. (5).

\[
a(\lambda, x) = \frac{6x(1-x)}{4(1-x^2)} \Lambda(\lambda, x) \tag{5}
\]

For given design \(C_L\) value(s) Eq. (5) can be used for defining the blade chord distribution which yields a given axial induction factor, while the blade twist distribution will derive from the flow angle \(\phi\) distribution, as given from the classical BEM velocity triangle,

\[
\tan \phi = \frac{(1 - x)}{x} \tag{6}
\]

by subtracting the design angle of attack (corresponding to the design \(C_L\) value(s)).

The power coefficient \(C_P\), rotor thrust coefficient \(C_T\) and aerodynamic bending moment coefficient \(C_{M_{\phi}}\) at arbitrary radial distance \(r\) are given by equations (7) to (9),

\[
\begin{align*}
C_p(\lambda) &= \int_{0}^{\frac{2\pi}{2}} \frac{8a(1-a)\lambda^{2} (1-x^{2})(1-s^{2})}{[1-(1-a)R^{2}+(1+\lambda^{2}+x^{2})]} ds \tag{7} \\
C_T &= \int_{0}^{\frac{2\pi}{2}} 8a(1-a)F \cos \phi ds \tag{8} \\
C_{M_{\phi}}(\phi) &= \frac{8a(1-a)^{2}}{B} \int_{0}^{\frac{2\pi}{2}} \frac{F \cos \phi}{(1-a)R + \lambda(1+\lambda^{2}+x^{2})} d\phi \tag{9}
\end{align*}
\]

where the above coefficients are defined as non-dimensional representations of the Power \((P)\), Thrust \((T)\) and Bending Moment \(M_{\phi}\) through Eqs. (10) to (12).
\[ P = \frac{1}{2} \rho R^2 V^4 C_p \quad (10) \]
\[ T = \frac{1}{2} \rho R^2 V^4 C_f \quad (11) \]
\[ M(\rho) = \frac{1}{2} \rho R^2 V^2 C_{M(0)} \quad (12) \]

3 Low Induction Rotors (LIRs)

For a pitching-variable speed HAWT design and for a given rotor radius the classical rotor aerodynamic design problem would seek to maximize the energy capture by maximizing the power coefficient \( C_P \). According to the BEM theory this would happen for an axial induction value \( \alpha = 1/3 \) and would correspond to a TSR design value \( \lambda \) which gets larger (along with \( C_{P,\text{MAX}} \)) as the aerodynamic performance of the blades \( k \) gets better (higher). As design \( \lambda \) increases the non-dimensional lift distribution \( \Lambda(\lambda, \alpha) \) gets thinner and, for the same family of blade profiles, the rotor solidity gets lower.

For a variable speed rotor, the design \( \lambda \) value (and therefore \( C_{P,\text{MAX}} \)) can be maintained over a range starting from a minimum wind speed, defined by the low-end capability of the variable-speed power conversion system, up to a maximum wind speed which is limited by the rotor maximum tip-speed, either for restraining noise or centrifugal loading. We shall call this maximum wind speed, where \( C_P = C_{P,\text{MAX}} \), “design wind speed” usually, the pitch speed variable turbines have their design wind speed just below their rated speed.

The top plot of fig.1 presents a plot of \( C_P \) versus \( \alpha \) for a three-bladed rotor with profiles of \( k=100 \). The \( C_{P,\text{MAX}} \) value is 0.4966 corresponding to an \( \alpha \) value of 1/3.

Suppose that we have an initial wind turbine and rotor design and we want to add some freedom in our design by redesigning the rotor, letting its radius free, but respecting all turbine related constrains (the rated rotational speed, the rated wind speed and power, the hub loading etc). We will assume for simplicity that the new rotor will use the same family of airfoils (same \( k \), considering Reynolds number effects as secondary at the scale of our interest).

Let \( R_0 \) be the initial rotor radius and let subscript “0” denote the initial (or reference) design, the one with \( \alpha = 1/3 \) corresponding to maximum \( C_P \).

Figure 1: Plots of non-dimensional coefficients, candidates for blade optimization, versus axial induction coefficient \( \alpha \). Application of Eqs. (7) and (9) for \( B=3 \), \( k=100 \) and \( \lambda = 8.85 \) (providing the maximum \( C_P \) value =0.4966 for the selected \( B \) and \( k \) combination). The \( \alpha \) value corresponding to the maximum coefficient value is shown on the x-axis.

The new design problem is formulated as:

Maximize \([C_P(\lambda, \alpha).R^2]/[C_{P,0}(\lambda_0, \alpha_0).R_0^2]\),

Constrained by \([C_{M(0)}(\lambda, \alpha).R^2]/[C_{M(0)}(\lambda_0, \alpha_0).R_0^2]=1\)

That is: “maximize the power output up to the design wind speed without exceeding the initial aerodynamic root blade moment” (see Eqs. (10) and (12)). By eliminating the radius dependence the optimization problem (13) can be recast as:

Maximize \([C_P(\lambda, \alpha)/C_{M(0)}(\lambda, \alpha)^{2/3}]\) \quad (14)

The solution of the optimization problem (14) for \( \alpha \), given \( \lambda = \lambda_0 \), is shown in the bottom plot of fig.1. The resulting value is \( \alpha = 0.187 \) and although it has been calculated for a “wrong” \( \lambda \) (= \( \lambda_0 \)) it can be shown that the optimum \( \alpha \) solution for (14) is \( \lambda \) insensitive.

As a result, the optimal rotor will have a larger radius: \( R/R_0 = 1.136 \); will capture more energy: \([C_P(\lambda, \alpha).R^2]/[C_{P,0}(\lambda_0, \alpha_0).R_0^2] = 1.087 \); and will be
less loaded than the initial one (design $C_T$ and $C_{M(0)}$ will be smaller), operating at a lower axial induction value $a = 0.187$ instead of $a_0 = 0.33$. In other words, we sacrificed $C_P$ in order to increase energy capture with a larger rotor diameter, while maintaining the aerodynamic bending moments at their initial level. This is feasible thanks to the special shape of the $C_P$ and $C_{M(0)}$ curves, where moving a little left from the optimum $a$ at the $C_P$ curve the power coefficient loss is milder than the corresponding reduction of bending moments in the $C_M$ plot.

The obvious question is “Is that cost effective, as long as we have a 13% longer blade now”? Let’s try to quantify the extra cost. In a very primitive approach and taking only the aerodynamic moments into account, we can assume that the new blade can maintain the cross-sections of the initial blade as long as they have the structural strength to undertake the reference bending loads which are not altered. Thus, the new blade will result as a “prolongation” of the reference one to the new radius its cross sections being invariant in terms of the radius fraction $x$. This means that the weight and “cost” of the new blade will increase by a factor of $R/R_0$. This would result in an increase of the levelized cost of the blade component of 4.6% (13% additional cost, 8.7% more power). Nevertheless, and since the levelized cost of the rotor blades is a small fraction of the overall levelized cost of electricity, particularly offshore, the selection of a larger, less loaded rotor for offshore turbines seems cost effective.

Evidently, the new rotor will have a proportionally increased tip-speed and consequently noisier. This is no problem for far offshore development. In addition, the new rotor will operate better in a multiple raw offshore wind farm or cluster since lower axial induction factors imply lower thrust coefficients and, thus, less wake effects in benefit of the production and loading of downstream turbines. Wake effects will be investigated later on in this paper.

Going one step back, the following question needs to be answered: “is the idea of blade prolongation compatible with the reduction of the axial induction?” We used Eqs. (5) and (6) to produce the planform characteristics of the new blade and compare them against the initial one. It is seen that the new rotor has significantly less non-dimensional lift than the initial one especially outboard. The trend is therefore right. In addition, no significant variation is seen in the flow angle plot and, therefore, the blade twist distribution. Prolongation can be done with the initial twist.

Figure 2: Characteristic distributions of the new blade (continuous line) against the reference blade ones (dashed line). Up-down: non-dimensional lift, flow angle, aerodynamic...
moment coefficient and peripheral induction coefficient.

As a final test we compare the two rotors on the wind turbine over its entire operating regime. In fig.3 we present the rotational speed schedule, the power coefficients and the power curves of the two configurations. Clearly, the new turbine is operating at lower \( C_{P,\text{MAX}} \) values over the entire wind speed regime. As expected the power capture benefits are limited at the below rated wind speeds regime. For a Class I site (mean annual wind speed 10 m/s, Weibull shape factor = 2) the annual energy capture of the new rotor drops to 3% (from 8.7%).

In the above presentation we suggested that optimization with constrained aerodynamic bending moment leads to solutions where the blade weight / cost are proportional to its length. Therefore, it looks tempting to define another optimization problem aiming at minimizing the levelized cost of the rotor itself rather than maximizing its power output. This new problem can be formulated as:

**Maximize**

\[
\frac{[C_P(\lambda, a)R]}{[C_{P0}(\lambda_0, a_0)R_0]}
\]

(15)

**Constrained by**

\[
\frac{[C_{M0}(\lambda, a)R^3]}{[C_{M0}(\lambda_0, a_0)R_0^3]} = 1
\]

Note that the term \([C_P(\lambda, a)R]\) represents the inverse of the levelized cost of blades and derives from the division of Power production \(\propto [C_P(\lambda, a)R^2]\) by Cost \(\propto R\). Eliminating the radius dependence as we did earlier Eq. (15) takes the form:

**Maximize** \(\{ C_P(\lambda, a)/ C_{M0}(\lambda, a)^{1/3} \}\)  (16)

The solution of the new optimization problem is also shown in fig.1. The optimal value of \(a\) is now 0.274, closer to \(a=1/3\) corresponding to the solution for maximum \(C_P\).

The optimal rotor will have a larger radius: \(R/R_0 = 1.039\). It will also capture more energy: \([C_P(\lambda, a)R^2]/[C_{P0}(\lambda_0, a_0)R_0^2] = 1.055\). The levelized cost will take its minimum at: \([C_P(\lambda, a)R]/[C_{P0}(\lambda_0, a_0)R_0]\)^{-1} = 0.984.

Over the entire power curve for the same Class I wind climate as before the annual energy capture gain will now be 2% instead of 3%. Although it is not clear whether the solution of optimization problem (15) gives more or less promising results than that of (13), it is clear that the combination of reducing rotor loading / axial induction from 1/3 to a value in the range [0.19 – 0.28] and increasing rotor diameter can be beneficial for reducing the cost of wind energy, in particular offshore.
Figure 4: Characteristic properties of rotors with the same root bending moment designed for different values of the axial induction factor. Plots are presented for the rotor diameter (D), the power production at design wind speed P (V\text{des}) the levelized rotor cost (LCE) and the annual energy production AnEP. All properties are divided by their corresponding reference values (a=1/3).

4 LIRs in Large Wind Farms

We can now investigate the possible benefits of a less loaded turbine in a large offshore wind farm environment. We shall consider a 500 MW offshore wind farm consisting of 10X10 5 MW turbines of the two design alternatives of figure 3 (highly loaded vs less loaded). The distance between the turbines is used as a parameter varying from 5 to 8 Diameters of the initial (highly loaded, D = 125 m) turbine. The wind rose is considered directionally uniform. The wind speed distribution is of Rayleigh type with mean 10m/s.

Figure 5 presents the capacity factor and wake losses of all 100 turbines for 8D spacing. Calculations are performed with CRES’ in-house wind farm analysis tool [9]. It is seen that by using the less loaded – larger diameter – turbines, the wind farm capacity factor increases by nearly 6%. Looking carefully this 6% comes partly (3%) from the increased annual production of the larger diameter turbine (AnEP, figure 4) and partly (another 3% roughly) from the reduction of the wake losses (second part of figure 5) due to the lower axial induction and, therefore, thrust coefficients of the larger rotors.

Figure 5: Capacity factor and wake losses per turbine in a 10X10 offshore wind farm for 8D spacing. Red dots refer to the initial turbines (highly loaded) and blue squares to the less loaded turbines. The dashed red and the blue line correspond to the wind farm mean values.

The effect of the turbines spacing on the annual energy production of the wind farm is presented in fig.6.

The 6% gain in annual energy production is more or less flat and independent from the turbine spacing. Evidently, this is very much connected to the accuracy of wake calculation in multi-raw offshore wind farms with engineering models, like the one used.
Figure 6: Annual energy production of a 10X10 offshore wind farm as a function of turbines spacing (x times Diameter). Red dots concern the initial turbines (highly loaded) and blue squares the less loaded turbines.

5 Conclusions

We investigated in some depth the impact of low induction rotor designs to the cost of offshore wind energy. We first demonstrated that a low induction high swept area rotor can capture more energy than a conventional design that aims at $C_{P,\text{MAX}}$, without a burden on the aerodynamic loading of the blades. Then we estimated the additional cost of the larger rotor and concluded that this cost is smaller than the expected benefit in energy production, especially offshore where the capital cost of the rotor corresponds to a small fraction (around 5%) of the levelised cost of wind electricity. This analysis was first done in stand-alone operation. Moving to the wind farm level we anticipated additional benefits for the low induction rotor designs in terms of energy capture and wake losses (also related to extra fatigue loading).

Closing, we want to make clear that this paper does not produce an actual design. It should be conceived as a first attempt for understanding possible “non-conventional” concepts that might be beneficial for the design of large offshore rotors. The methodology used and the assumptions made regarding the rotor and wind turbine loading are therefore pretty crude. For instance, in our analysis we have had no discussion on extreme loads, fatigue loads, blade-tower clearance issues etc.

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References